Filter Topologies with Minimum Peak Stored Energy

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Abstract— **In this paper the question of enhancing the power handling capability of resonator filters by chosing a suitable filter topology which maximally reduces the peak time averaged stored energy (t.a.s.e.) is addressed. A well known network transformation that alters the filter topology but leaves the 2-port parameter of the network unchanged is employed. After defining the cost function a stochastical search method is used to find the global minimum. Results presented include Butterworth filter topologies of degree 2 to 5.**

I. Introduction

Multipaction and dielectric breakdown continues to be a problem in cavity filters and multiplexers in some radar and communication applications. This is due to the large build-up of the electric field strength in in some of the filter cavities. A measure of the electromagnetic field strength is the time averaged stored energy (t.a.s.e.) and it has been shown [1], [2] that the t.a.s.e. is proportional to the group-delay of Chebychev and Butterworth filters. When this result is combined with the well-known relation between selectivity and group-delay [3], [4] the sharp rise of the t.a.s.e. for frequencies around the passband cut-off frequencies [5] and hence the large electric field strengths [6] at these frequencies in microwave direct-coupled cavity filters can be explained. Recently [5], [7] the t.a.s.e. distributions in 3 t.c., 5 extracted pole and a q.c. realisation of the same power transfer characteristic of a highly selective minimum phase filter were compared with each other. It was reported that the peak t.a.s.e. in the different realisations varied significantly of up to 2.48 times. This rises the question which realisation exhibits a maximally reduced peak t.a.s.e.. The question of obtaining such "optimum" filter topologies is addressed in this work. When fromulated as an optimisation problem the cost function exhibits multiple local minima. Thus, a stochastical search method is employed to carry out the search for the global minimum of the cost function. Due to the limited computing power available, only Butterworth filters of degree 2 to 5 are considered. Optimum topologies with a maximally reduced peak t.a.s.e. presented include 2^{nd} and 3^{rd} degree low-pass Butterworth filters.

II. Optimisation and Results

The ladder network in Fig. 1 is a popular realisation of Butterworth all pole filters. The peak stored energy for degrees 2 up to 5 is found in Tab. I. In [5], [7] it was reported that different realisation result in different t.a.s.e. distributions and hence the question rises whether there are Butterworth filter topologies with a lower peak t.a.s.e. than the popular ladder network. Employing a rotational network transformation [8], [9], which has been widely used in the synthesis of general multiple-coupled resonator filters leaving the 2-port parameters of the transformed network unchanged, a class of multipath networks can be generated with the same power transfer function. When the rotation angles are chosen as optimisation parameters those filter topologies can be found that exhibit the smallest peak stored energy in any of the resonators. This corresponds to minimising the following cost function in the frequency range $\omega \in]-\infty, \infty[$

$$f_{cost}(\phi_1, \ldots, \phi_m, \omega) = \max\left\{\max\left\{W_2(\omega)\right\}, \ldots, \max\left\{W_{n+1}(\omega)\right\}\right\} \quad (1)$$

where n is the degree of the filter, m is the number optimisation parameters

$$m = \frac{n(n-1)}{2} \quad (2)$$

and W_r is the stored energy in the resonator at the r^{th} node.

In the 2^{nd} degree case there is only one optimisation parameter and the optimisation problem can be solved graphically. In Fig. 3 the peak t.a.s.e. in the two resonators is shown. The cost function is the maximum

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of the two functions. The minimum of the maximum of these two functions occurs at 0, 90, and 180 degrees and corresponds to a ladder network. Hence, the ladder network is the optimum network in this case.

In the case of higher degree networks there are 3 or more optimisation parameters and the optimisation problem can not be solved graphically. An additional difficulty is the fact that the cost function has multiple local minima and maxima which has proven to make it impossible to employ fast gradient-type optimisation engines. Instead a simulted annealing optimisation engine was employed [10], [11] to carry out the minimisation. Also, it may be shown that it is sufficient to only consider rotation angles in the range from 0 deg to 90 deg. This reduces the parameter space considerably. The simulated annealing optimistion engine was tested successfully on the second degree case. The resulting optimum topology for a 3^{rd} degree Butterworth filter is shown in Fig. 4. The peak t.a.s.e. (Tab. I) in the filter is reduced from $1.333\,[Joule]$ to $1.1442\,[Joule]$ which corresponds to an improvement of about 17%. It is interesting to note that the peak t.a.s.e. in all of the resonators is identical. This was not the case for the 2^{nd} degree Butterworth filter. Also, the optimisation engine never produced any other topology other than the one in Fig. 4 so it can be concluded that this is the only optimum topology.

In the case of 4^{th} degree topologies with a Butterworth characteristic two optimum topologies with the same peak t.a.s.e. were found by the stochastical search. The improvement over the ladder network in this case is about 34% and again the peak t.a.s.e. in all resonators is identical.

When the stochastical search is applied to 5^{th} degree Butterworth filter topologies an improvement over the ladder network of about 43% can be achieved.

III. Conclusion

In this paper the question of finding filter topologies with maximally reduced peak stored energy, and hence enhanced power handling capability, has been addressed for the first time. A stochastical search method was employed in order to find the global minimum of the maximum of the peak stored energy in the resonators of 2^{nd}, 3^{rd}, 4^{th} and 5^{th} degree Butterworth filters and to handle the many local minima and maxima of the cost function. Optimum topologies of a class of multi-path networks of 2^{nd} and 3^{rd} degree Butterworth filters have been presented. It is interesting to note that the peak t.a.s.e. in the optimum topologies is significantly lower than in the ladder network but they do not achieve the theoretical lowest peak t.a.s.e., the average peak total t.a.s.e.

$$W_{tot,av} = \frac{\max\{W_{tot}\}}{n} \tag{3}$$

where $\max\{W_{tot}\}$ is the peak total t.a.s.e.. and n is the degree of the filter.

Applying the optimisation engine to Chebychev filters of the same degree it can be shown that the optimum filter topologies obtained have the same coupling arrangement.

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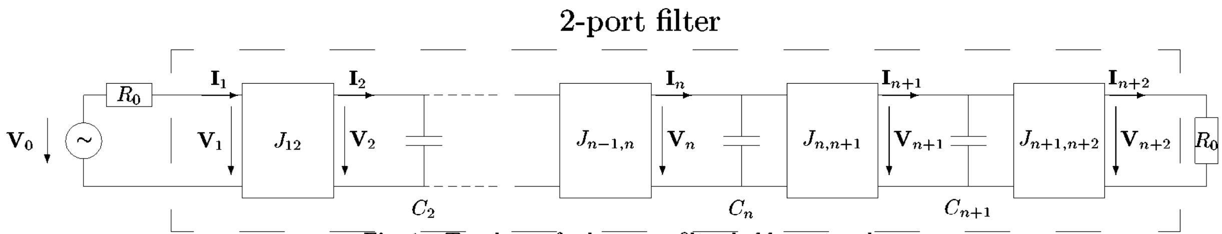


Fig. 1. Topology of a low-pass filter ladder network.

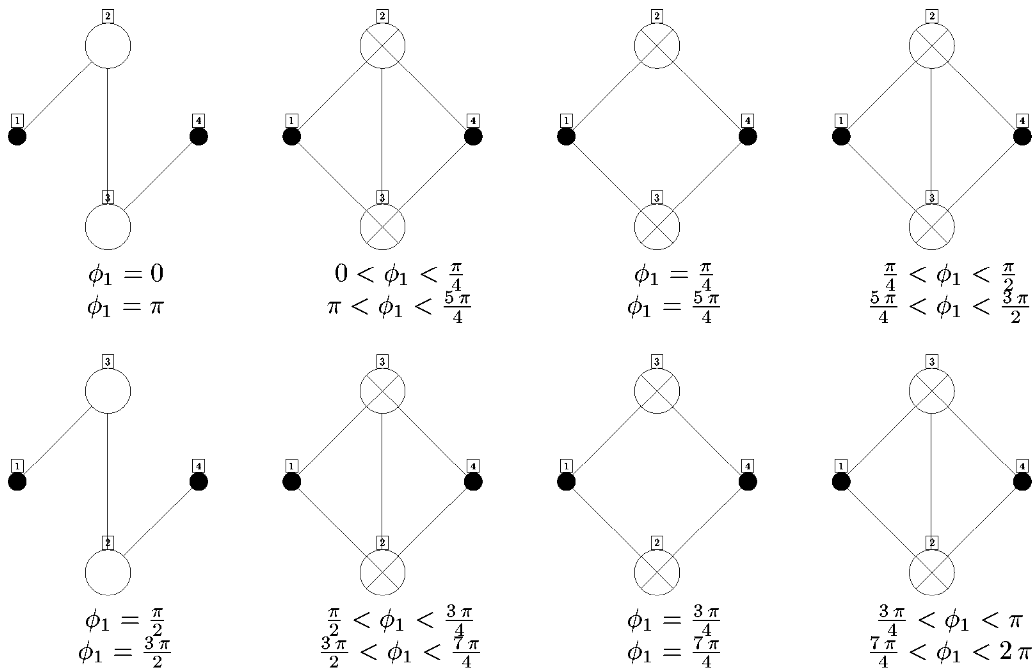


Fig. 2. Filter topologies of 2^{nd} degree Butterworth filters.

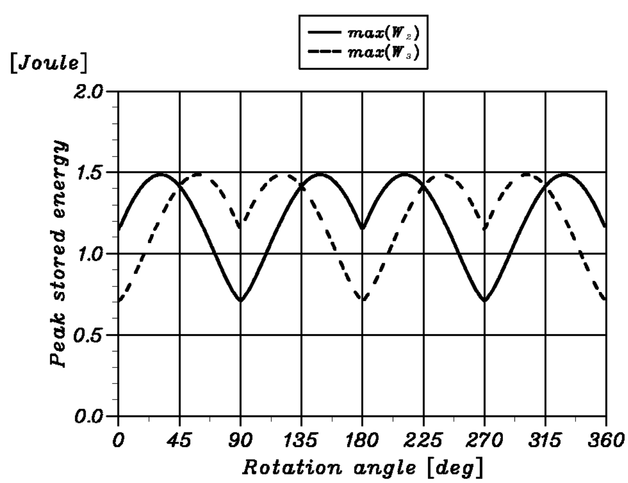
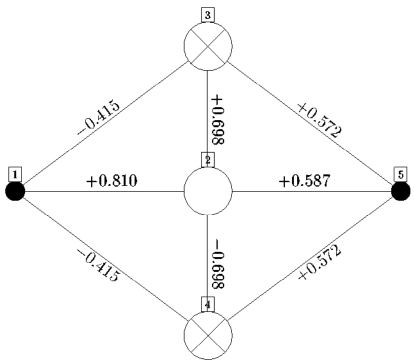


Fig. 3. Maximum stored energy in the two resonators of a 2^{nd} degree Butterworth filter which is driven by a generator with available power of 1 Watt



(a) Network topology

Plane rotation	Rotation angle [rad]
$2-3$	0.000000 ± 10^{-6}
$2-4$	0.627374 ± 10^{-6}
$3-4$	0.785398 ± 10^{-6}

(b) Rotation angles with respect to the ladder network

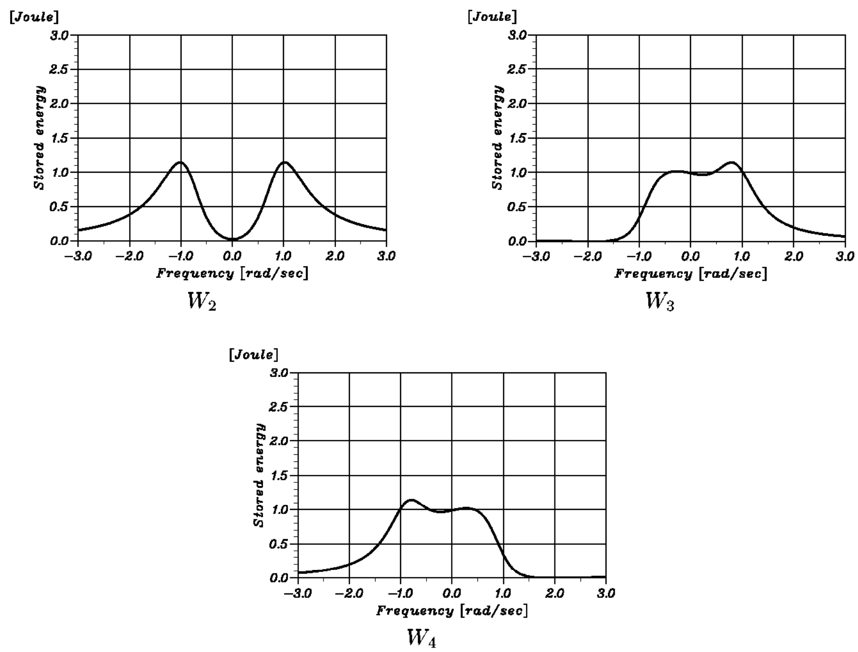


Fig. 4. Stored energy distribution in the optimum 3^{rd} degree Butterworth filter driven by a generator at node 1 with available power of 1 Watt.

Degree	Ladder network	Optimised topology	Average total peak t.a.s.e.
1	1.0000	1.0000	1.0000
2	1.1441	1.1441	0.8536
3	1.3333	1.1442	0.9143
4	1.8029	1.2039	0.9774
5	2.1862	1.2430	1.0331

TABLE I

COMPARISON OF MAXIMUM PEAK STORED ENERGIES IN BUTTERWORTH FILTERS ($P_A = 1\,W$). THE AVERAGE TOTAL PEAK T.A.S.E. IS DEFINED IN EQUATION (3).